Title: Far Light, Star Bright?

Brief Overview:

This activity has been developed to provide opportunities for students to investigate relationships between light intensity and distance with possible applications to mass and distance relationships. Students will have the opportunity to determine the type of regression equation that will best-fit data that has been collected, and use this equation to make predictions about various mathematical relationships. Communication and graphing skills, use of technology, and team work are incorporated throughout this activity.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  The students will apply power functions to investigate the relationship between light intensity and distance.

- **Mathematics as Communication**
  The students will express mathematical ideas orally and in writing.

- **Mathematics as Reasoning**
  The student will predict graphical data and compare to results obtained through CBL usage.

- **Mathematical Connections**
  The student will make applications between algebraic concepts and related topics in science.

- **Algebra**
  The student will represent and analyze collected data from tables and graphs to determine the curve of best fit.

- **Functions**
  The student will analyze and make predictions to determine the type of function represented by their collected data.

Links to Maryland High School Mathematics Core Learning Goals:

1.1.2: The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

1.2.4: The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

3.1.1: The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

3.2.1: The student will make informed decisions and predictions based upon the results of simulations and data from research.

3.2.2: The student will make predictions by finding and using a line of best fit and by using a given of best fit.

Links to Maryland High School Science Core Learning Goals:

1.3: The student will carry out scientific investigations effectively and employ the instruments, systems of measurement, and materials of science appropriately.

1.4: The student will demonstrate that data analysis is a vital aspect of the process of scientific inquiry and communication.

1.5: The student will use appropriate methods for communicating in writing and orally the processes and results of scientific investigation.

1.6: The student will use mathematical processes.

2.1.2: The student will describe current efforts and technologies used to study the universe.

5.5.1: The student will cite evidence of the quantum nature of matter and its applications.

Grade/Level:

Grades 9-12; Algebra I, Algebra II, Pre-Calculus; Earth Science, Physical Science, Physics, Astronomy.

Duration/Length:

Two to three 50-minute or one to two 90-minute classes.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Using the graphing calculator and the CBL system properly
- Representing collected data using appropriate tables and graphs
- Finding the curve of best fit using the graphing calculator

Objectives:

Students will:

- collect, graph, and analyze data using the TI-82/83 graphing calculator and the CBL light probe.
- explain the relationship between light intensity and distance using the inverse square law.

Materials/Resources/Printed Materials:

- TI Real-World Math Bulb Program
- TI-82/83 graphing calculator
- CBL system and light probe
- Metric ruler
- Light sources with disk
- Wood Block
- Tape
- Pencil

Development/Procedures:

- This lesson helps students identify the relationship between light intensity and distance using the Inverse Square Law, which is a power function. It will enable students to determine which type of equation will best-fit their data, and use this equation to make predictions about inverse relationships and patterns in the world around them.

- The teacher needs to familiarize the students with following key terms and topics:

    - light intensity
    - inverse-square law
    - exponential, power, and linear functions
    - best-fit regression equations
    - star brightness/distance comparisons

- A teacher resource, to explain the inverse-square law with light, using a graph paper grid, is available through the Project Star: The Universe in Your Hands, Kendall Hunt Publishers, 1993 copyright.

- Prior lessons reviewing the TI-82/83 Graphing Calculator and the use of the CBL, specifically the light probe, will be needed before beginning the activity.

- The teacher will need to obtain a graph-link to download the BULB program from the Real-World Math with the CBL System Activity Booklet, Texas Instruments, 1994.

- CBL systems and graphing calculators for groups of 2-4 students will be needed. Blocks of wood are necessary to raise the light probe to receive the direct light from the light source. Students may need to use books to raise the light source to the correct height. Prior to the lab, disks to cover the diameter of the light source, with an opening for light to be emitted, should be made from black construction paper.

Disk Example

- The Activity Sheet: Finding the "Best-Fit" Regression Equation, has been provided to assist the teacher and the student with the necessary keystroke order.

ACTIVITY SHEET:
FINDING THE "BEST-FIT" REGRESSION EQUATION

Using your TI Graphing Calculator, follow the steps below to find the best-fit regression equation.

1. Press **STAT >** to highlight **CALC.**

2. Choose from the different types of regression equations. Press either **5,** or **A,** or **B**. (This will give you a LINEAR, POWER, OR EXPONENTIAL regression)**.**

3. Press the following keys in order: **2nd , 1 , comma key , 2nd , 2 ,** to make the equation from the correct lists (**L1, L2**).

4. Press **ENTER.** <u>The a, b, and r values will appear on the screen.</u>

The r value is the correlation coefficient. This value indicates how good the linear regression model fits your data. A perfect fit has a r value of 1 or -1.

5. Press **Y= , then CLEAR.**

6. Press the following in order: **VARS , 5(to Statistics) > , >** to EQ. Press **7** for REG EQ. This step puts the regression equation into Y1**.****

****In <u>step 6,</u> you may change Y1 to Y2 or Y3... in order to see more than 1 type of equation at a time.**

7. Press **GRAPH** to see your regression equation.

8. Repeat steps 1-7 above to fit the other regression equations. When you find the equation that best-fits, that which has a value for r closest to 1 or -1, you will have found the best regression equation for your data.

INVESTIGATION: FAR LIGHT, STAR BRIGHT?

Have you ever imagined how information that we cannot get to is possibly measured? How did man know that it was possible to land on the moon before we landed on its surface? How have we measured the distances to far away objects in our Universe, using only equipment found on the Earth or on satellites above the Earth's atmosphere?

In this investigation, you will use technology that will enable you collect and analyze light intensity versus distance. This activity will simulate experiments that various astronomical unmanned satellites utilize while in space.

Purpose:

Upon completion of this investigation, the student will be able to:

- collect, graph, and analyze data using the TI-82/83 graphing calculator and the CBL light probe.
- explain the relationship between light intensity and distance using the inverse square law.
- explain the relationship between the color of an object and its reflectivity.

Materials:

- TI-82/83 graphing calculator
- CBL system and light probe
- Metric ruler
- Light sources
- TI-Real World with the CBL System Bulb Program
- Tape
- Wood block
- Pencil

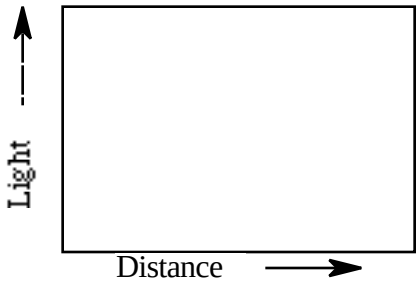
Prediction:

1. Using your pre-lab notes, predict how light intensity changes as distance from the object changes. Use the space below to describe the relationship between distance and light intensity as you move away from the glowing object.

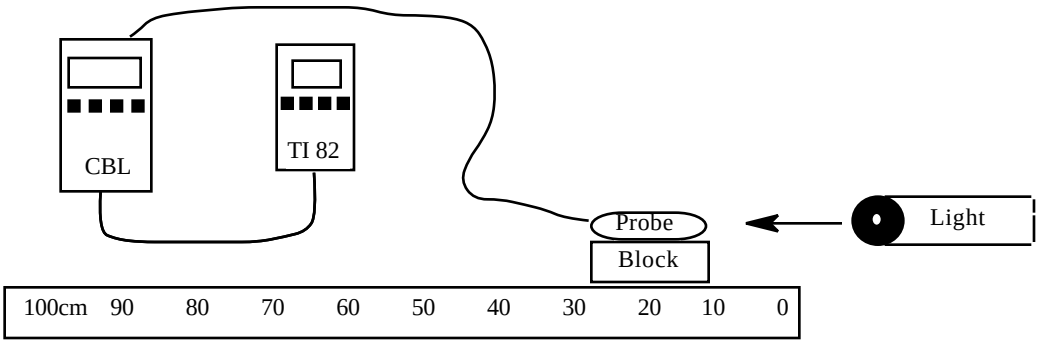
   _____

   _____

2. Using your written prediction above, sketch your predicted graph in the box provided.



3. In your prediction, what represents...
   a) the independent variable, light or distance? _____
   b) the dependent variable, light or distance? _____

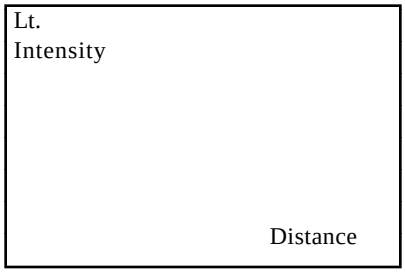


Procedure:

1. Following your teachers instructions and the diagram above, set up the lab equipment as shown. The light probe will be plugged into Channel 1 of the CBL System. Connect the CBL System to the TI Graphing Calculator using the link.
2. The light probe should be fastened securely to the block of wood using tape. Position the light probe on the 10 cm marking of the meter stick, as shown. Turn on your light source and aim the beam of light directly at the light probe. If necessary, use books to raise the light source to the proper height.
3. Turn on the TI Graphing Calculator. Press **PRGM,** select the program **BULB**, then press **ENTER** again to receive the *Light At a Distance Program*. Follow directions to options and select **COLLECT DATA**. Follow the directions as listed, making sure that all connections between the CBL, TI Graphing Calculator, and the Light Probe are secure.
4. IMPORTANT NOTE: For all trials, substitute .1 M for 1.0 M, as listed in the first measurement for distance. (On the meter stick, 10 cm = .1 M).
5. Turn out the lights in the classroom.
6. After the initial measurement, move the light probe, which is attached to the wood block, in increments of 10 cm. For example, your first measurement will be taken at 10 cm, the second at 20 cm, the third at 30 cm, etc. until the program ends and data collection is complete(graph will appear on screen).
7. Record your results from your graph and data, located in the lists, in the Data/Analysis Section of this investigation. Have your teacher check your results, for accuracy, before proceeding with the analysis questions.

Data/Analysis: Answer all questions completely.

1. Record your data from the investigation in the graph and table provided. Use appropriate labels where needed. Repeat the investigation if there are errors with your results.



Distance	Lt. Intensity
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	
0.9	
1.0	

2. Using your graph and data, describe the relationship between light intensity and increasing distance in complete sentences.

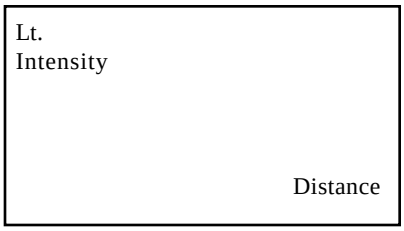
_____

_____

3. Mathematicians and scientists often attempt to find the line, or curve, of "best fit" for their collected data. This "best fit" line is called a regression line or curve. In this section, you will use your collected data to determine if your data best-fits a linear, exponential or power regression equation. Use prior knowledge, or the directions on the Best-Fit Equation Activity Worksheet provided by your teacher, to determine the correct answer. Record your results below.

a. What type of regression best-fits your data?_____
b. Write the regression equation in the space provided.

c. Sketch the graph showing this regression equation from your screen in the box provided below.



d. What is your correlation coefficient or **r** value?_____
e. How did you know that your value for this correlation coefficient, gave you the best-fit equation for your data? Explain your answer.

4. a. Using your data and the equation from question 2, determine the intensity of light for the following distances:

Distance	**Light Intensity**
20 cm	_____
50 cm	_____

b. Explain how you can determine the value for the light intensity to a distance which you did not measure?

_____

_____

c. Use this information to determine the light intensity of an object which is 150 cm away. Explain how you received your answer. Answer = _____

_____

_____

5. How would astronomers use this relationship to predict the distances to stellar, or star-like, objects in space?

_____

_____

6. What is the closest star to the Earth? _____
Is this object really the brightest object in our Universe? Explain your answer using your findings from this investigation.

_____

_____

7. Another class performed this investigation, but each group had a light source that varied in size and strength. Compare and contrast the similarities and differences that would result.

_____

_____

_____

_____

Conclusion:

- Review your predictions that were made at the beginning of the investigation. Compare your initial predictions with your lab results.

_____

_____

_____

_____

- After reviewing your results, explain what you have learned from this investigation and how this information can be applied to real-life situations.

_____

_____

_____

Extension/Follow Up:

- Have students use different sources of light and make predictions about the similarities and differences in the lab outcome.
- Have students research other similar regression equations, such as mass and distance relationships, and set up possible lab activities to simulate the investigation.
- Research how astronomers find the distances to stellar objects, including galaxies, using the Inverse Square Law. Discuss how scientists must make allowances for great distances, using the apparent magnitude and absolute magnitude for stars.

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Performance Assessment:

Teacher's Guide: Far Light, Star Bright?

Introduction

Objectives Covered: This task assumes your students have received instruction and been assessed on the concepts and/or objectives reviewing the Inverse Square Law and Mathematical Correlation's.

- Analyze and identify simulated data between light intensity and distance using the inverse square law.
- Find the curve of best fit.
- Explain the relationship between light intensity and distance using the inverse square law.

This task is to be done as an independent activity. You will be reading a set of general instructions and then allowing students to work individually at their own pace. Make any accommodations for students.

Tools/Materials Needed for Assessment

- Student response sheet
- Pencil
- Calculator(optional)

Administering the Assessment

This task should take approximately 50 minutes to administer.

Distribute a copy of the assessment to each student.

SAY: *Today you are going to use mathematics and science skills to complete some items and activities in this assessment related to our unit on the Inverse Square Law. You will complete everything on your own. Take time now to skim through the different parts of the assessment.*

Pause to allow students to look through the assessment. Have the students write their names on the first page.

SAY: *Are their any questions? You may begin.*

When students have completed the assessment, collect their materials.

Performance Assessment: Far Light, Star Bright?

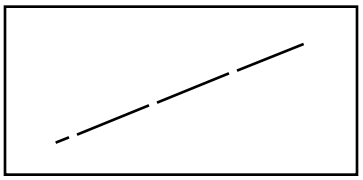
Name : _____

1. In a line of best fit the ideal data would have points which :

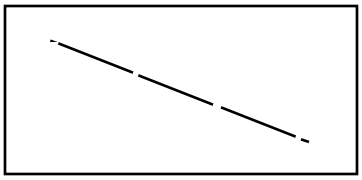
A. are equally spaced
B. include every point
C. include the first and last points
D. pass through the point of origin

2a. Which of the following graphs best represents the inverse square law ?

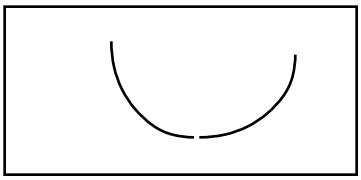
A.



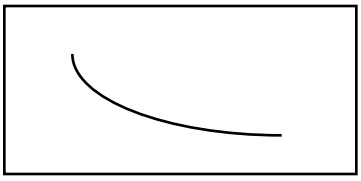
C.



B.



D.



2b. Explain why you selected the graph.

3. Which equation best represents a power function ?

A. $y = 6 - x$
B. $y = x^2 + 3$
C. $y = ax^b$
D. $y = (3x)$

4. According to the inverse square law $I = K/x^2$. What is the intensity if $K = 1$ and $x = 5$?

A. 0.4000
B. 0.0625
C. 0.0400
D. 0.0200

5. The inverse square law could be used to explain the weightlessness of astronauts in space. The formula is $W = K/D^2$. Where W = weight on the surface of the earth, D = distance from the center of the earth and K is the constant. An astronaut weighs 200 lb. on the surface of the earth. The distance from the surface of the earth to the center of the earth is 4,000 miles.

a. Select the correct value for the constant K.

A. 82.0×10^{-4}
B. 3.20×10^{9}
C. 8.00×10^{4}
D. 3.20×10^{-9}

b. Show your work in the space provided to support your selection.

6a. Assuming that the constant K is 4×10^{8} calculate the weight of an astronaut 14, 000 miles from the center of the earth. Select the correct response.

A. 0.204 lb.
B. 200.4 lb.
C. 2.040 lb.
D. 20.40 lb.

b. Show your work in the space provided to support your selection.

7. Use the following data answer the following questions.

Intensity in milliwatts	Distance in Meters
1.00	1.0
0.25	2.0
0.06	4.0
0.04	5.0
0.01	10.0

a. Based on the pattern shown in the above data, label the axis and sketch the predicted curve.

b. Select the general mathematical formula which best represents the data.

A. $I = 1/X$
B. $I = 1/X^3$
C. $I = 1/X^2$
D. $I = X^2$

8. You are on a darkened football field, that is marked in 10 yd. increments, on your bicycle with a single headlight. The following materials are available for your use :

- TI Graphing Calculator
- CBL Unit with cable
- Light probe

Using the information obtained through the light activity that you performed in class, explain a simulation of the inverse square law. Describe the starting location of the bicycle and the probe. Include directions or steps you would follow to determine four values for the intensity and distance.

Performance Assessment: Far Light, Star Bright?

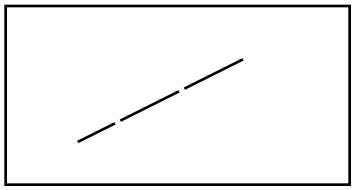
Scoring Guide

1. In a line of best fit the ideal data would have points which : (1 pt.)

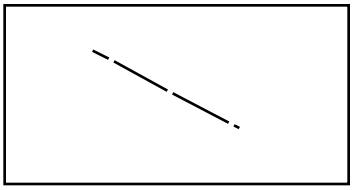
A.are equally spaced
B. include every point
C. include the first and last points
D. pass through the point of origin

2a. Which of the following graphs best represents the inverse square law ? (1 pt.)

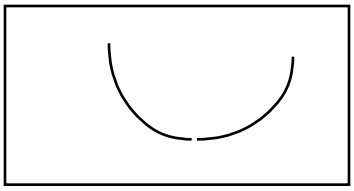
A.



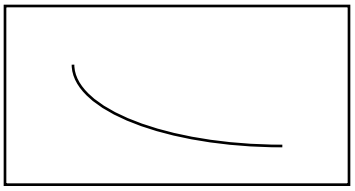
C.



B.



D.



2b. Explain why you selected the graph. (1 pt.)

Graph D shows a power function or inverse square law.

3. Which equation best represents a power function ? (1 pt.)

A. $y = 6 - x$
B. $y = x^2 + 3$
C. $y = ax^b$
D. $y = (3x)$

4. According to the inverse square law $I = K/x^2$. What is the intensity if $K = 1$ and $x = 5$? (1 pt.)

A. 0.4000
B. 0.0625
C. 0.0400
D. 0.0200

5. The inverse square law could be used to explain the weightlessness of astronauts in space. The formula is $W = K/D^2$. Where W = weight on the surface of the earth, D = distance from the center of the earth and K is the constant. An astronaut weighs 200 lb. on the surface of the earth. The distance from the surface of the earth to the center of the earth is 4,000 miles.

a. Select the correct value for the constant K. (1 pt.)

A. 82.0×10^{-4}
B. 3.2×10^{9}
C. 8.0×10^{4}
D. 3.2×10^{-9}

b. Show your work in the space provided to support your selection.

$200 = K / (4{,}000)^2$
$K = (200)(4{,}000)^2$
$K = 3{,}200{,}000{,}000$ or
$K = 3.2 \times 10^{9}$

(2 pts. - as shown)
(1 pt. - correct substitution, wrong answer, e.g., $200 = K/(4{,}000)^2$ K = 3,200)
(0 pt. - wrong substitution, no work)

6a. Assuming that the constant K is 4×10^{8} calculate the weight of an astronaut 14, 000 miles from the center of the earth. Select the correct response.

A. 0.204 lb.
B. 200.4 lb.
C. 2.040 lb.
D. 20.40lb.

b. Show your work in the space provided to support your selection.

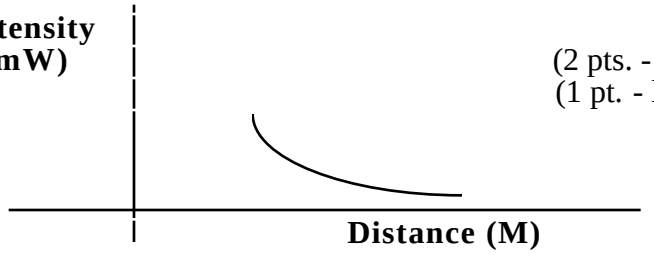
$W = (4 \times 10^{8}) / (14{,}000)^2$
$W = 2.04$

(2 pts. - as shown)
(1 pt. - incorrect substitution, e.g., $W = (14{,}000)^2 / (4 \times 10^{8})$, W = 0.49)
(0 pt. - incorrect work or no work)

7. Use the following data to answer the following questions.

Intensity in milliwatts	Distance in Meters
1.00	1.0
0.25	2.0
0.06	4.0
0.04	5.0
0.01	10.0

a. Based on the pattern shown in the above data, label the axis and sketch the predicted curve.



(2 pts. - label and sketch as shown)
(1 pt. - label or sketch)

b. Select the general mathematical formula which best represents the data.

A. $I = 1/X$
C. $I = 1/X^2$
B. $I = 1/X^3$
D. $I = X^2$

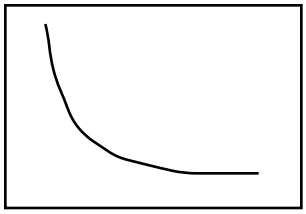
8. You are on a darkened football field, that is marked in 10 yd. increments, on your bicycle with a single headlight. The following materials are available for your use :

- TI Graphing Calculator
- CBL Unit with cable
- Light probe.

Using the information obtained through the light activity that you performed in class, explain a simulation of the inverse square law. Describe the starting location of the bicycle and the probe. Include directions or steps you would follow to determine four values for the intensity and distance.

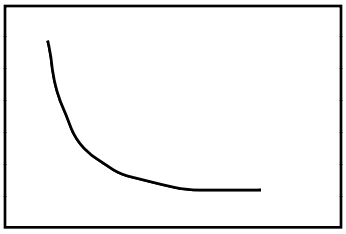
Far Light, Star Bright? Answer Guide to Investigation

Prediction:

1. The correct answer should be that light intensity decreases as the distance increases.
2. 

3. a. distance b. light

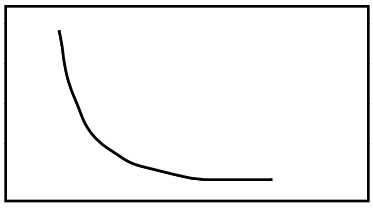
Data/Analysis:

1. 

Sample Data

Distance	Light Intensity
0.1	0.14957
0.2	0.07316
0.3	0.05498
0.4	0.03718
0.5	0.02494
0.6	0.02160
0.7	0.01826
0.8	0.01493
0.9	0.01270
1.0	0.01047

2. As distance increases, light intensity decreases.

3. a. power regression
   b. $y = 0.108x^{-3.66}$
   c. 

d. -0.98

e. This answer was closest to -1 than other regression equation attempts.

4. 20 cm = 0.07316
   50 cm = 0.02494

b. Use your regression equation to make predictions that are not on the graph.

c. $y = 0.108x^{-3}$
substitute 150 for x
answer = 3.2×10^{-8}

5. The farther the distance in space, the intensity of the light decreases.

6. Sun, this is not the brightest star in our galaxy. The sun is closer, therefore it appears very bright. Alpha Centauri is one of the brightest stars in our galaxy.

   The relationship will continue to be an inverse square relationship, but the light intensity will vary according to the wattage of the light source used.

Rubric Scoring Guide: Far Light, Star Bright?

4 - State that the bicycle is placed on a marked yard line.
State that the light probe is located at least 10 yards away from the bicycle.
State that the light probe is moved in equal increments away from the bicycle at least four times.
State that at each move the data would be collected.

3 - State that the bicycle is placed on a marked yard line.
State that the light probe is located away from the bicycle, not necessarily at least 10 yards.
State that the light probe is moved in any increment of yards away from the bicycle at least four times.
State that at each move the data would be collected.

2 - State that the bicycle is placed on a marked yard line.
State that the light probe is located away from the bicycle.
State that the light probe is moved away from the bicycle.
No mention is made of data collection.

1 - State that the bicycle is placed on a marked yard line.
State that the light probe is located away from the bicycle.

0 - Any other response.